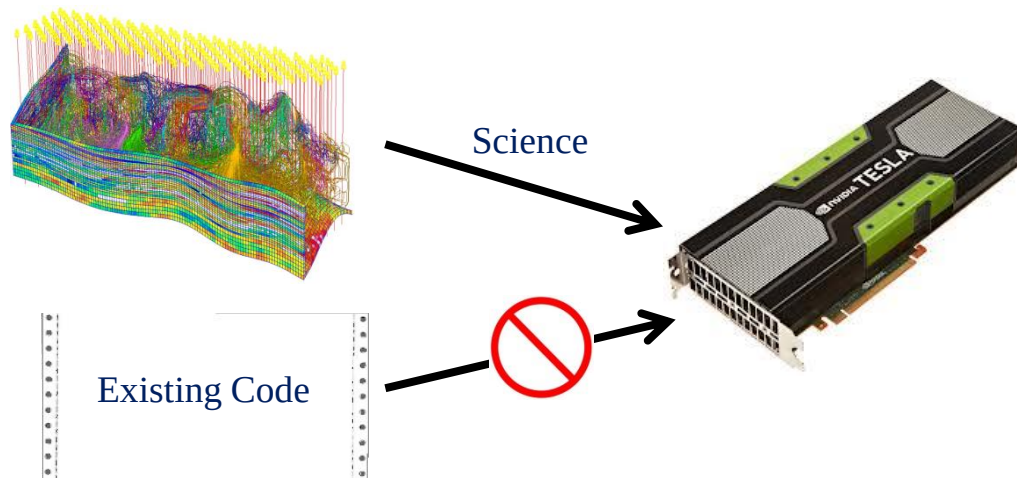


HP-CAST, June 21, 2014

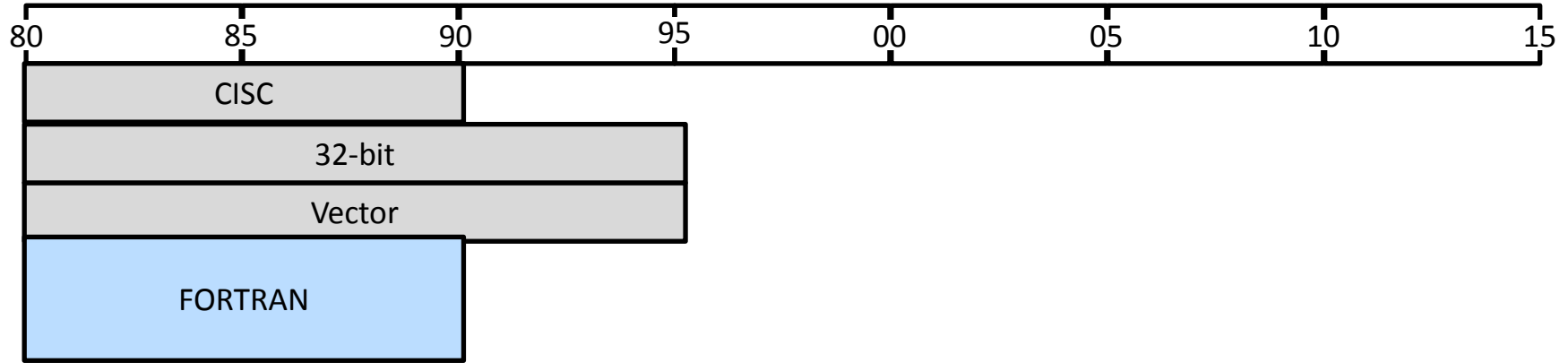
Dr. Ronald Young

Port the Science, Not the Code.

Application and library requirements for maximum accelerator performance.



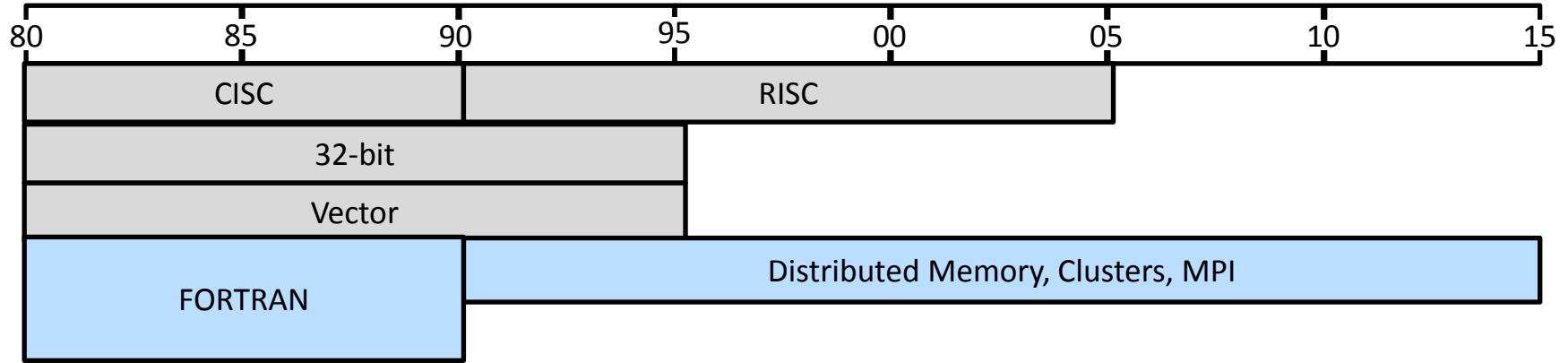
Hardware & Software History



Processor Speed $\sim$ Memory Speed



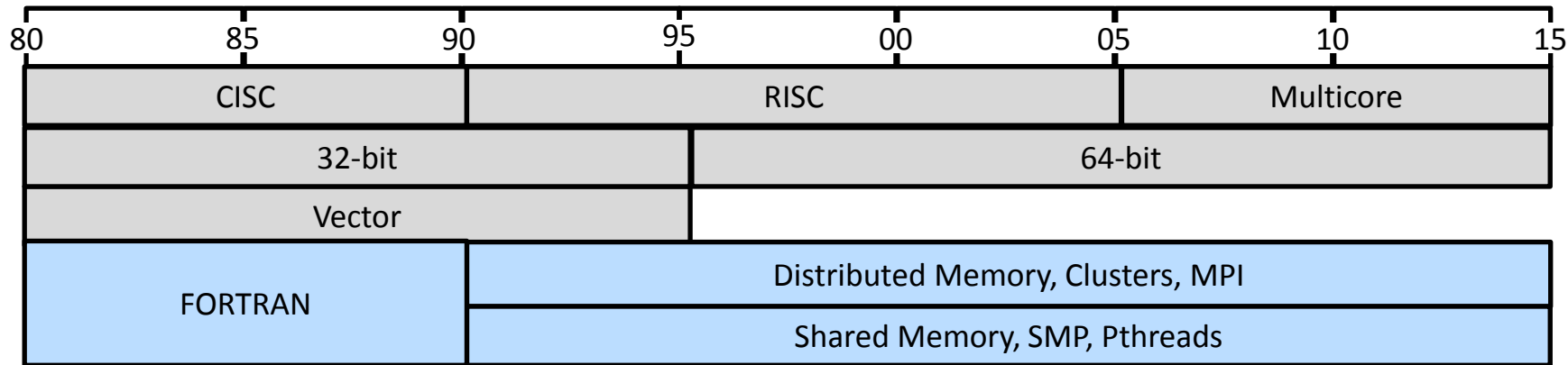
Hardware & Software History



Processor on a single chip
Lower cost/calculation
Potential for increasing processor speed



Hardware & Software History



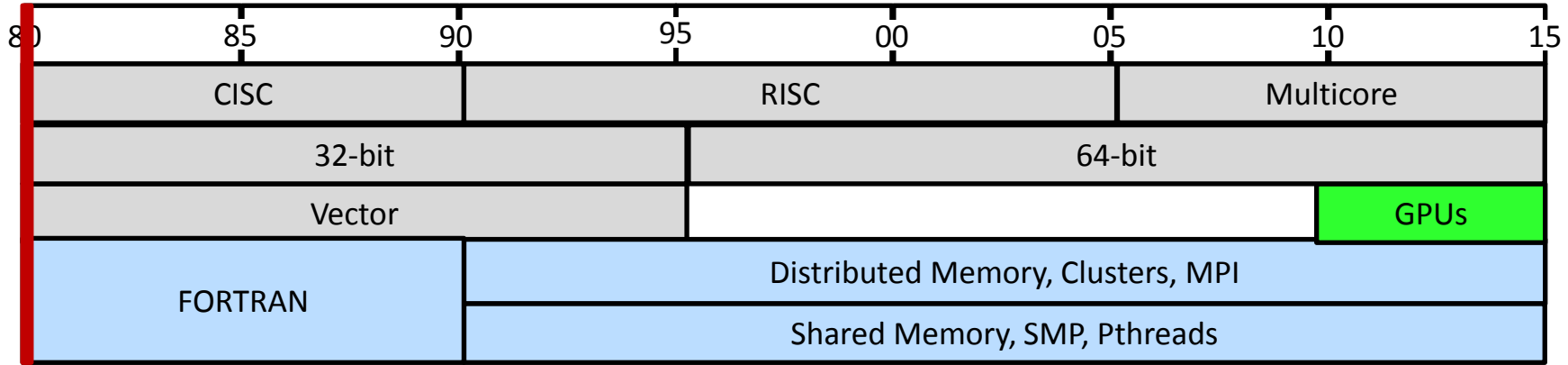
2 programming models emerge:

Distributed memory

Shared memory

64-bit addressing becomes available

Hardware & Software History



Floating point limited

86,000,000 Times Faster
166,000,000 Times Cheaper



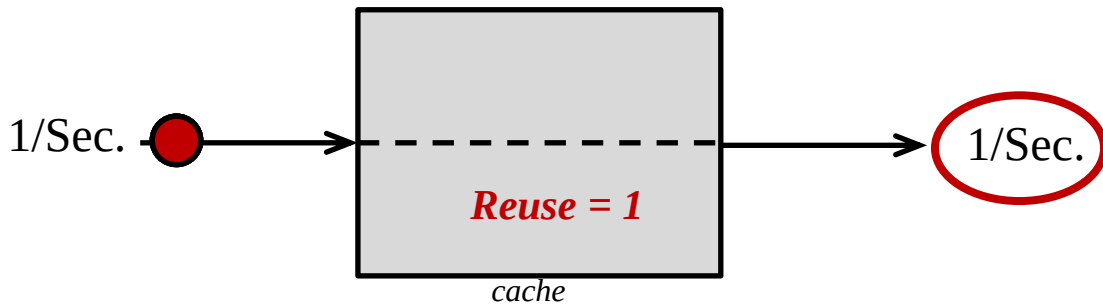
Data limited

When was your software designed?

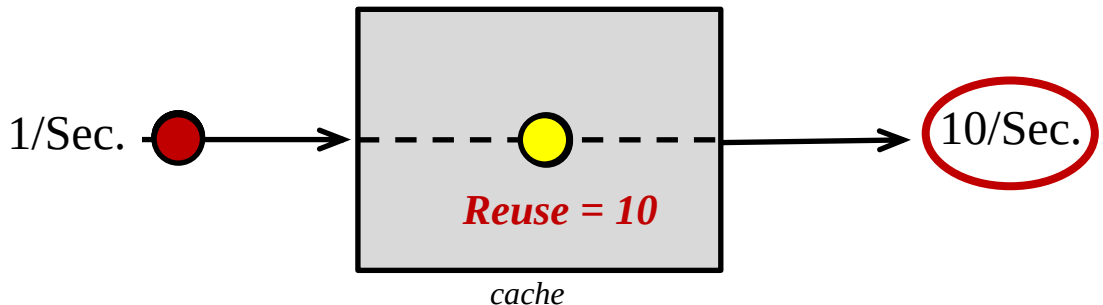
Reuse *Efficiently linking slower input to faster requirement*

- Reuse = Number of times a data element is used in a computation.

Old machines: Low Reuse



New machines: High Reuse



BLAS 1 Reuse

When $M=N=K$

$$\begin{bmatrix} \bigcirc C_{ij} \end{bmatrix} = \begin{bmatrix} \text{---} \end{bmatrix} \begin{bmatrix} | \end{bmatrix}$$

$C(N,N) \quad A(N,N) \quad B(N,N)$

```

DO 30 J=1,N
  DO 20 I=1,M
    DO 10 L=1,K
      C(I,J) = A(I,L)*B(L,J)
10    CONTINUE
20  CONTINUE
30  CONTINUE
    
```

BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1

$$\frac{\text{Flops}}{\text{Data}} = \frac{2N}{(2N+1)} \sim 1$$

BLAS 2 Reuse

$$\begin{bmatrix} | \\ \hline \end{bmatrix} = \begin{bmatrix} \square \\ \hline \end{bmatrix} \begin{bmatrix} | \\ \hline \end{bmatrix}$$

$C(N,N) \quad A(N,N) \quad B(N,N)$

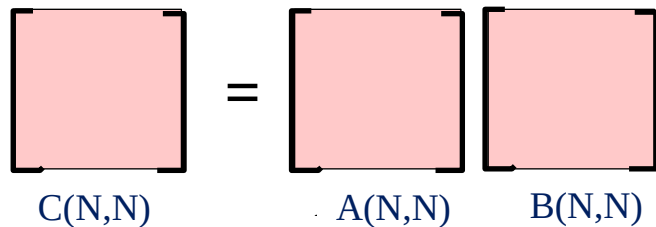
```

DO 30 J=1,N
  DO 20 I=1,M
    DO 10 L=1,K
      C(I,J) = A(I,L)*B(L,J)
10    CONTINUE
20  CONTINUE
30 CONTINUE
    
```

BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2

$$\frac{\text{Flops}}{\text{Data}} = \frac{2N^2}{(N^2+2N)} \sim 2$$

BLAS 3 Reuse



```
DO 30 J=1,N
  DO 20 I=1,M
    DO 10 L=1,K
      C(I,J) = A(I,L)*B(L,J)
10    CONTINUE
20  CONTINUE
30  CONTINUE
```

BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2
3	DGEMM	$3N^2$	$2N^3$	$2N/3$

$$\frac{\text{Flops}}{\text{Data}} = \frac{2N^3}{3N^2} = \frac{2N}{3}$$

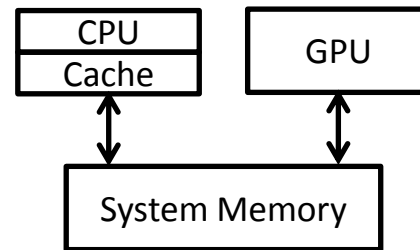
BLAS 3 Reuse

$$\begin{bmatrix} \end{bmatrix} = \begin{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix}$$

$C(N,N) \quad A(N,N) \quad B(N,N)$

```
DO 30 J=1,N
  DO 20 I=1,M
    DO 10 L=1,K
      C(I,J) = A(I,L)*B(L,J)
10    CONTINUE
20  CONTINUE
30  CONTINUE
```

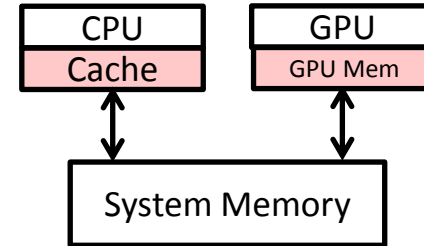
BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2
3	DGEMM	$3N^2$	$2N^3$	$2N/3$



Reuse – Small Data

$$\begin{bmatrix} \boxed{} \end{bmatrix}_{C(N,N)} = \begin{bmatrix} \boxed{} \end{bmatrix}_{A(N,N)} \begin{bmatrix} \boxed{} \end{bmatrix}_{B(N,N)}$$

BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2
3	DGEMM	$3N^2$	$2N^3$	$2N/3$

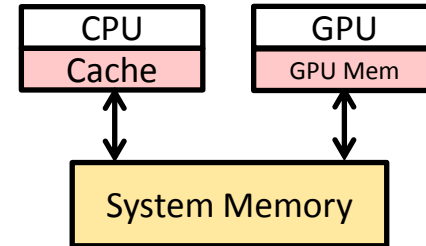


Reuse - More Data

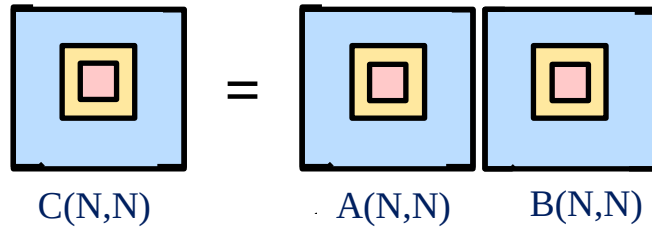
$$\begin{bmatrix} \boxed{\boxed{\text{C(N,N)}}} \end{bmatrix} = \begin{bmatrix} \boxed{\boxed{\text{A(N,N)}}} \end{bmatrix} \begin{bmatrix} \boxed{\boxed{\text{B(N,N)}}} \end{bmatrix}$$

$\text{C(N,N)} \quad \text{A(N,N)} \quad \text{B(N,N)}$

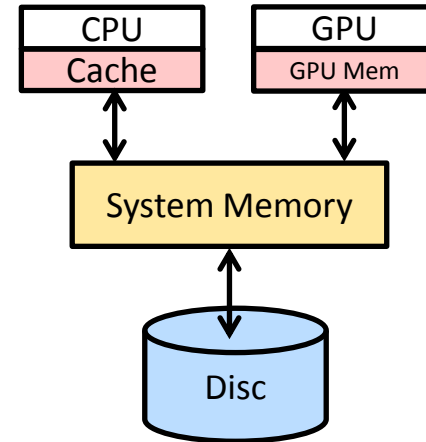
BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2
3	DGEMM	$3N^2$	$2N^3$	$2N/3$



Reuse – Big Data

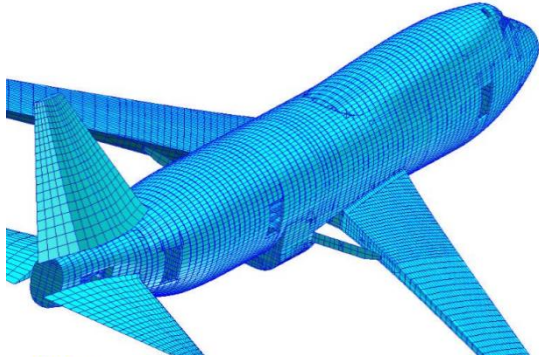


BLAS	Function	Data	Flops	Reuse
1	DDOT	$2N+1$	$2N$	1
2	DGEMV	N^2+2N	$2N^2$	2
3	DGEMM	$3N^2$	$2N^3$	$2N/3$



Application Development

Discretization



1. Domain is divided into a large number of pieces.
2. Equations are formed for each piece $\{a\}$, $\{b\}$
3. Pieces are assembled into a global system of equations

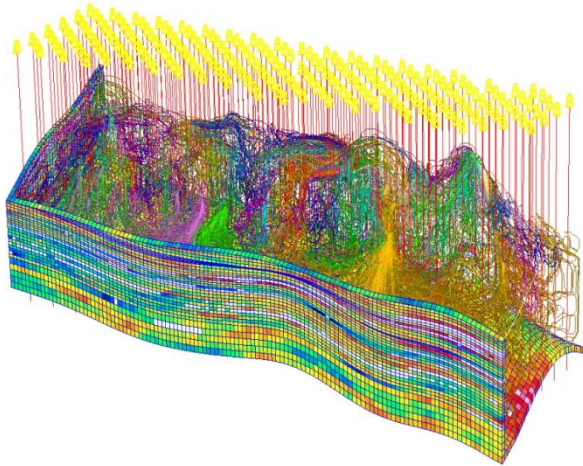
$$[A]\{X\} = \{B\}$$

4. The system is then solved for the solution $\{X\}$

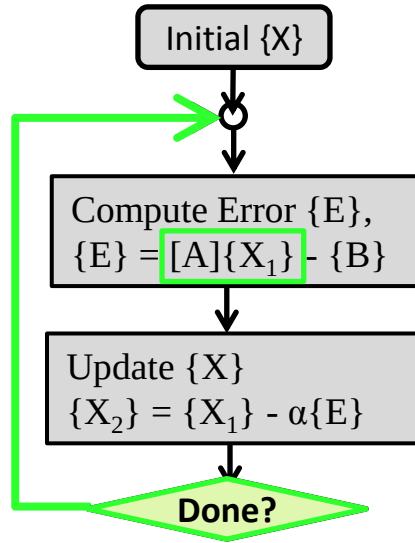
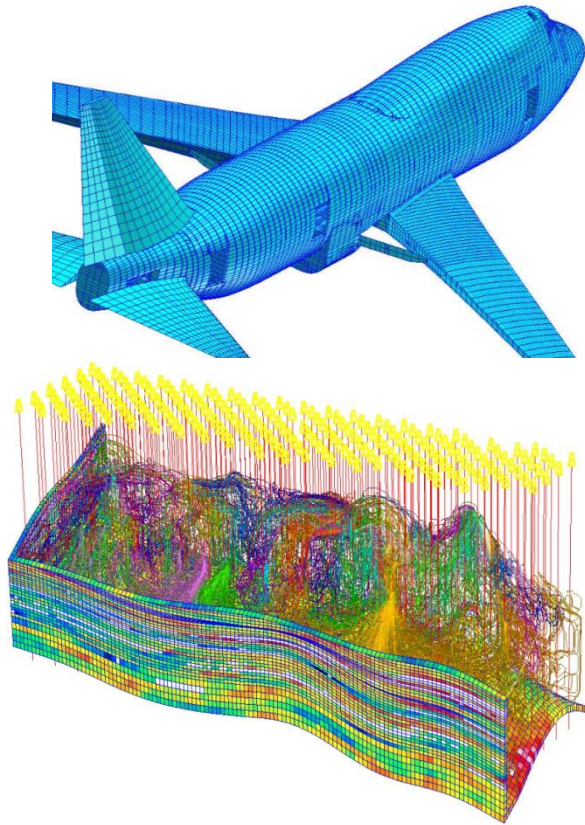
More pieces gives better answer

More pieces = more data and processing

$\therefore$ Machine (speed, storage) determines accuracy



Solving $[A]\{X\}=\{B\}$



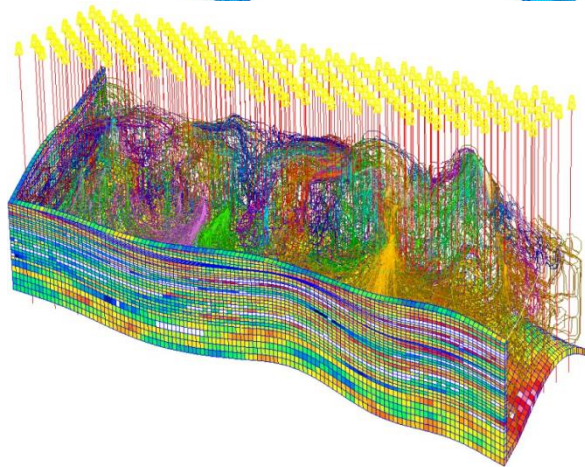
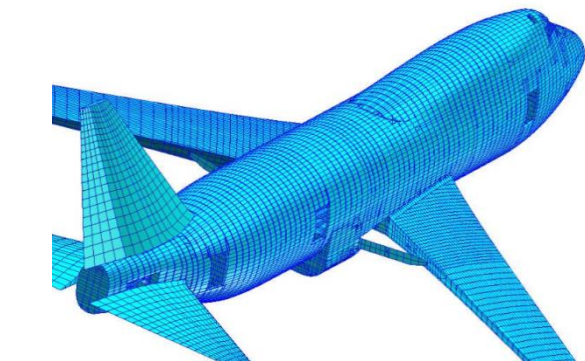
Explicitly

- (+) Minimum data
- (-) Low reuse: BLAS 2
- (-) Many iterations
- (-) Convergence risk
- (-) Data dependent
- (-) Unpredictable run times
- (-) Stability -> small increments
- (-) Multiple solutions
- (-) Indirect addressing

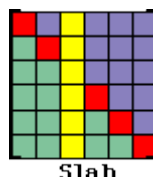
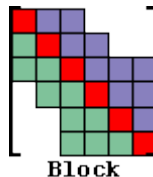
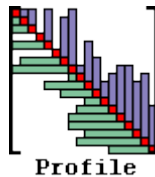
Expertise required for:

- Convergence criteria
- Selecting initial $\{X\}$
- Selecting α

Solving $[A]\{X\}=\{B\}$



Solve
 $[A]\{X\} = \{B\}$
Factor $[A]$
Solve for $\{X\}$



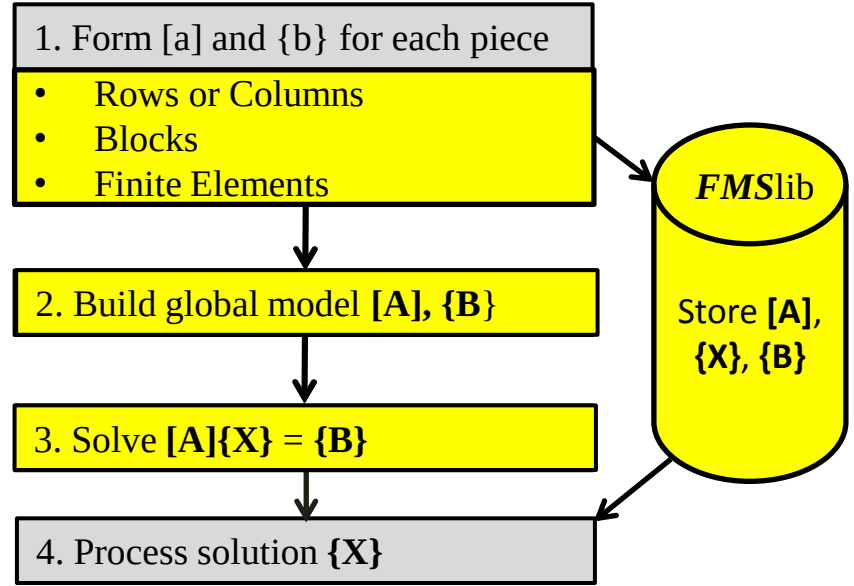
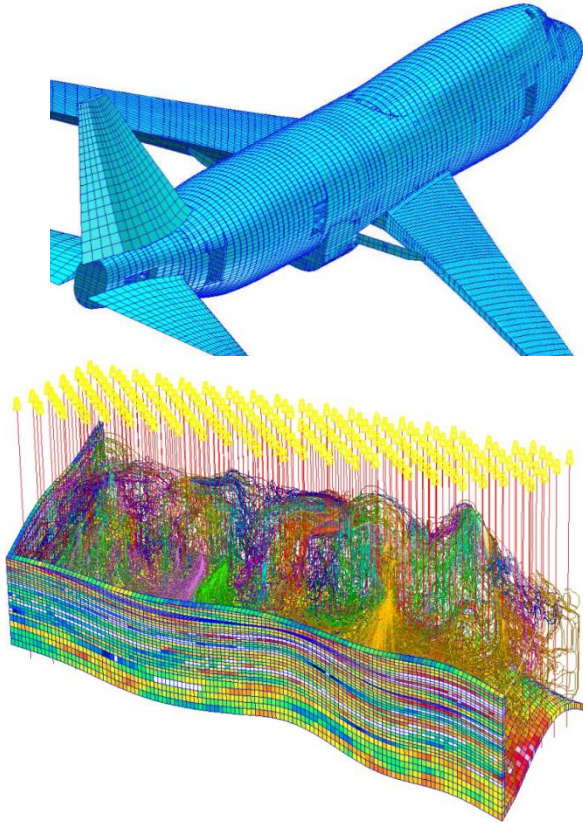
Implicitly

- (-) Fill data
- (+) High reuse: BLAS 3
- (+) One iteration
- (+) No convergence risk
- (+) Data independent
- (+) Predictable run times
- (+) Multiple solutions
- (+) Unconditionally stable
- (+) Incremental addressing

Additional advantages:

- Production applications
(no “expertise” required)
- Modern computers
(hierachical memory systems)

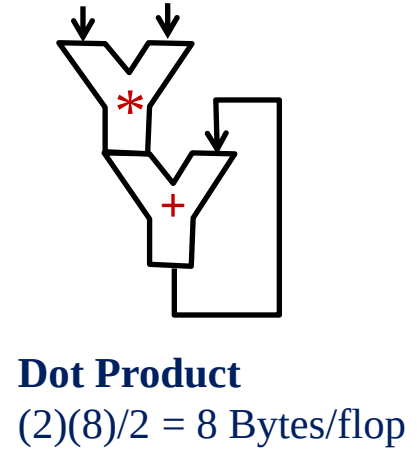
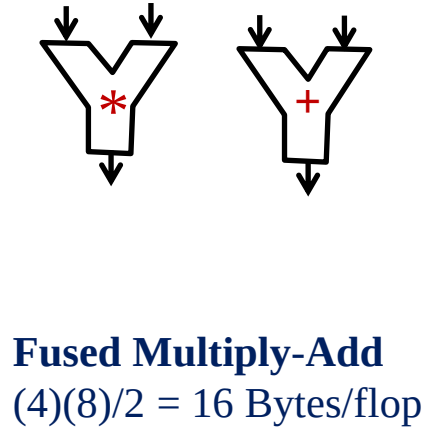
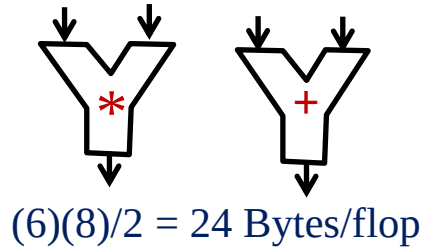
Solving $[A]\{X\}=\{B\}$ *Implicitly*



Explicit vs Implicit

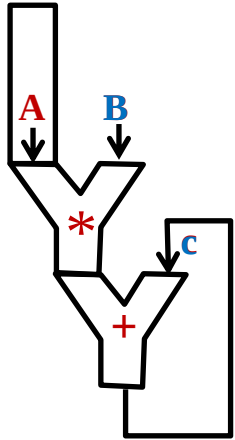
	Explicit	Implicit
Data	Minimum	+ Fill
Reuse	Low (2)	High (BLAS 3)
Iterations	Many	1
Convergence	Expert required	Automatic
Data Independent	No	Yes
Run times	Variable	Predictable
Multiple solutions	No	Yes
Stability	Small increments	Unconditional
Addressing	Slow (Indirect)	Fast (Incremental)

Hardware Reuse

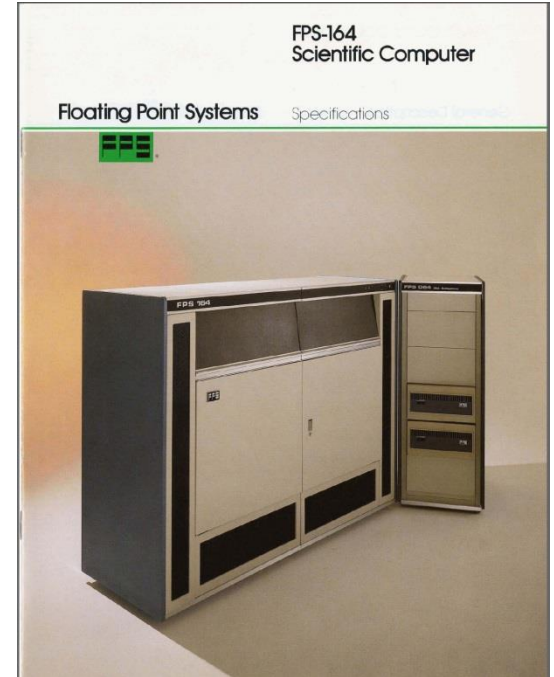


Hardware Reuse

$$\begin{bmatrix} \text{O} \\ \text{c} \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{{A}} \end{bmatrix} \begin{bmatrix} \text{---} \\ \text{{B}} \end{bmatrix}$$



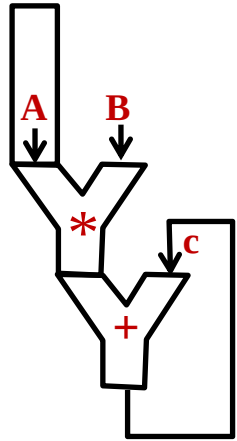
Vector Dot Product
(1)(8)/4 = 4 Bytes/flop



1981 Floating Point Systems 164
First 64-bit accelerator
First version of FMSlib

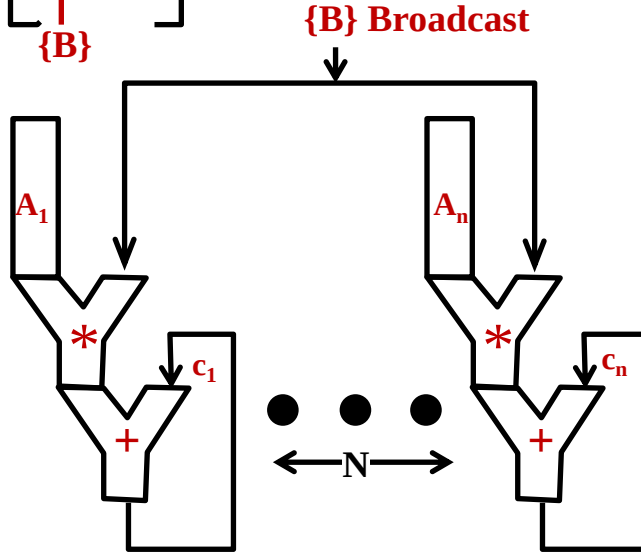
Hardware Reuse

$$\left[\begin{array}{c} \text{C}_1 \\ \vdots \\ \text{C}_n \\ \text{C} \end{array} \right] = \left[\begin{array}{c} \{A_1\} \\ \vdots \\ \{A_n\} \end{array} \right] \left[\begin{array}{c} | \\ \{B\} \end{array} \right]$$



Vector Dot Product

(1)(8)/4 = 4 Bytes/flop



Parallel Vector Dot Product

$(1/N)(4) = (4/N)$ Bytes/flop

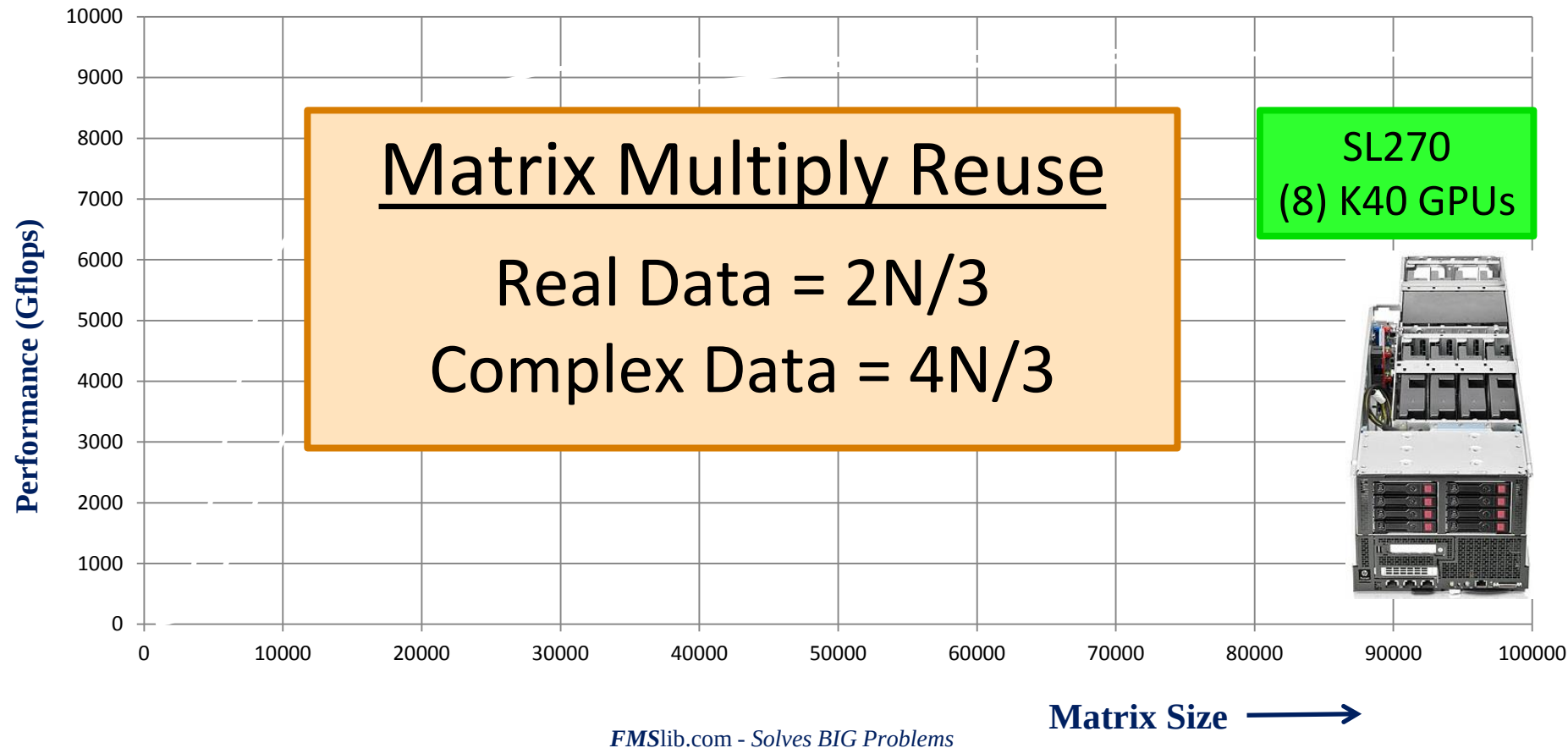


1985 FMSlib Accelerator

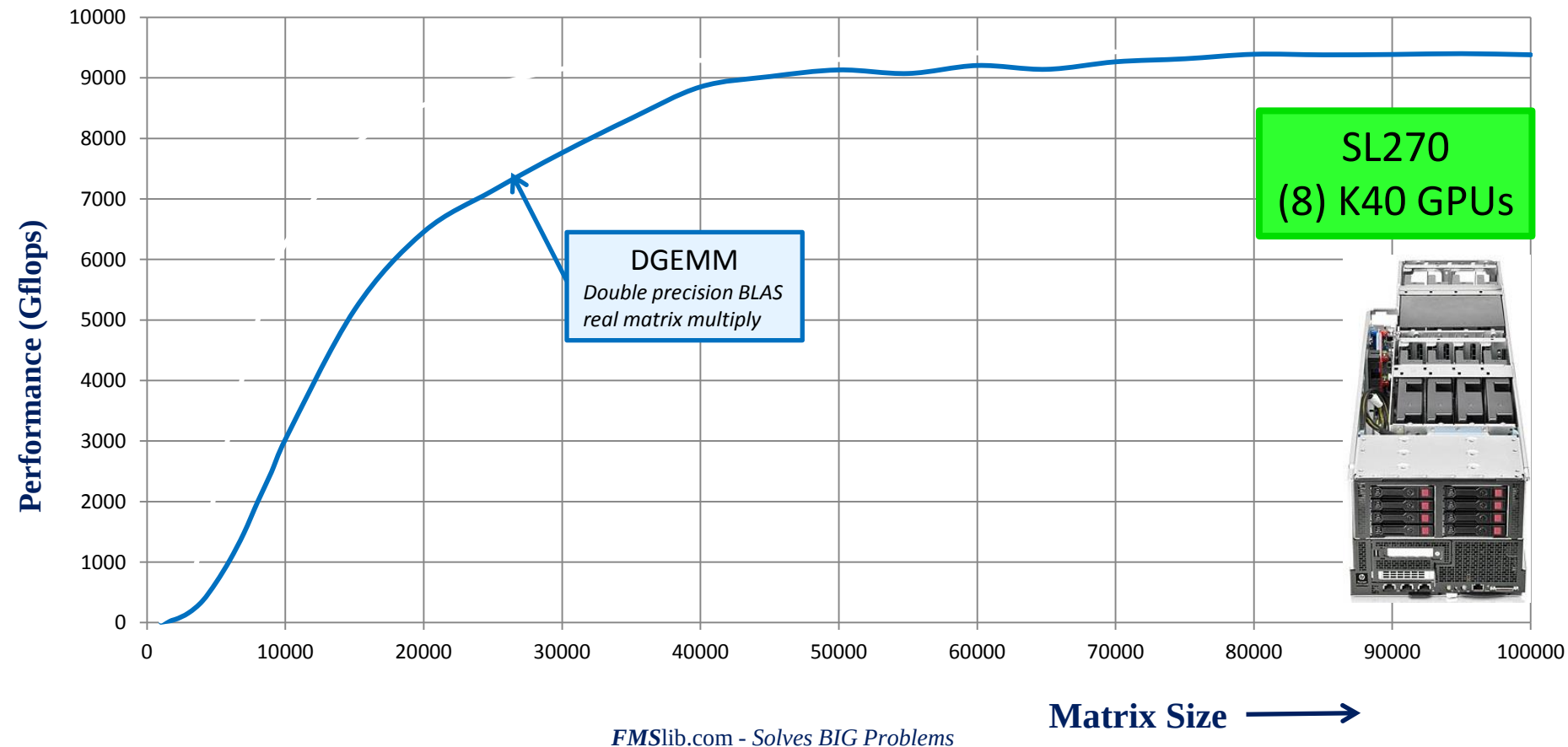
Hardware Reuse

Design	Bytes/Flop	<i>For 1 Tflop, Data Rate required (Tbytes/sec.)</i>	Data Rate in (Gbytes/Sec)	Reuse Required
General Multiplier, Adder	24	24	8	3000
Fused Multiply-Add	16	16	8	2000
Dot Product	8	8	8	1000
Vector Dot Product	4	4	8	500
N Parallel Vector Dot Product	4/N	4/N	8	500/N

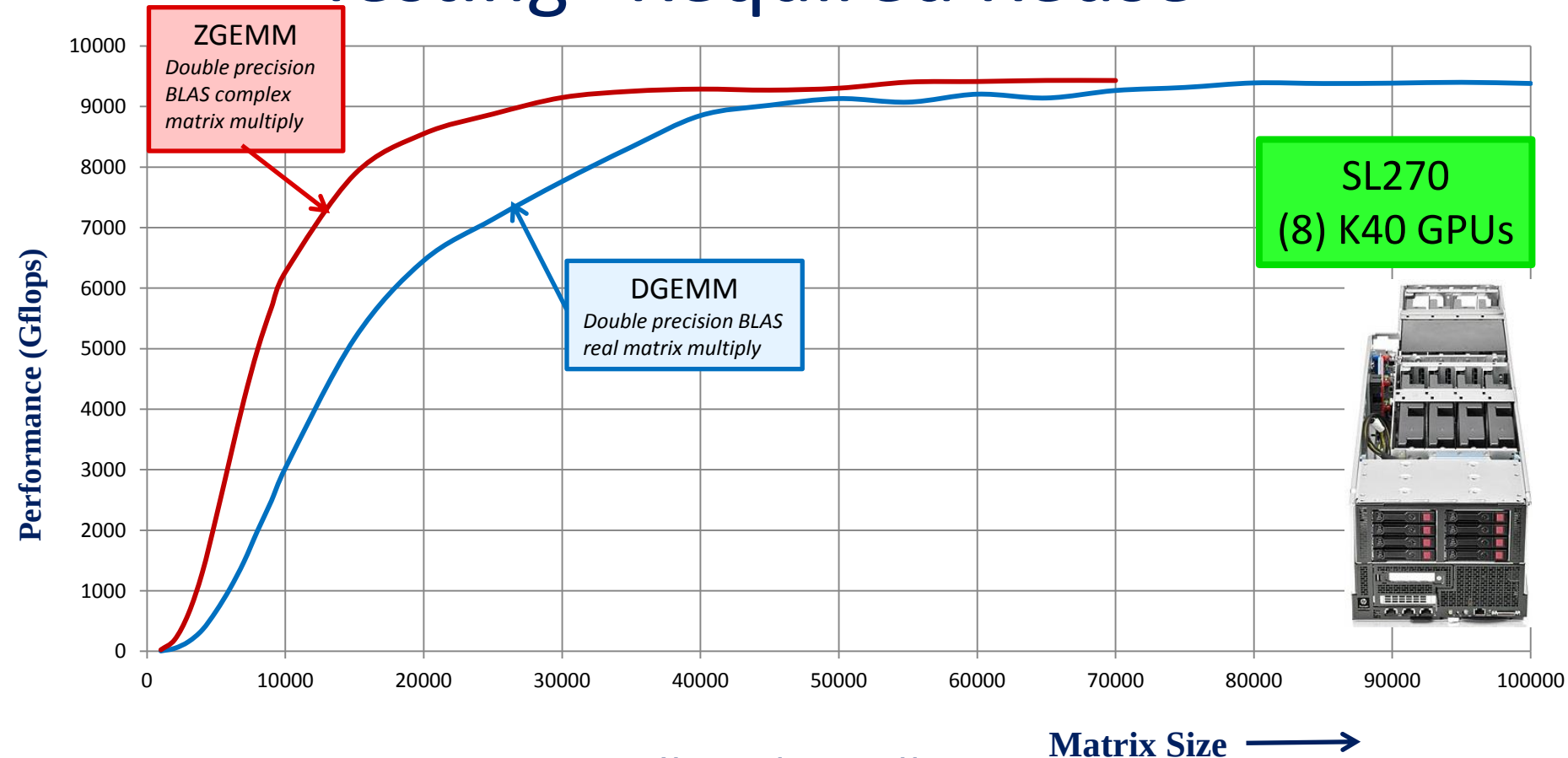
Testing “Required Reuse”



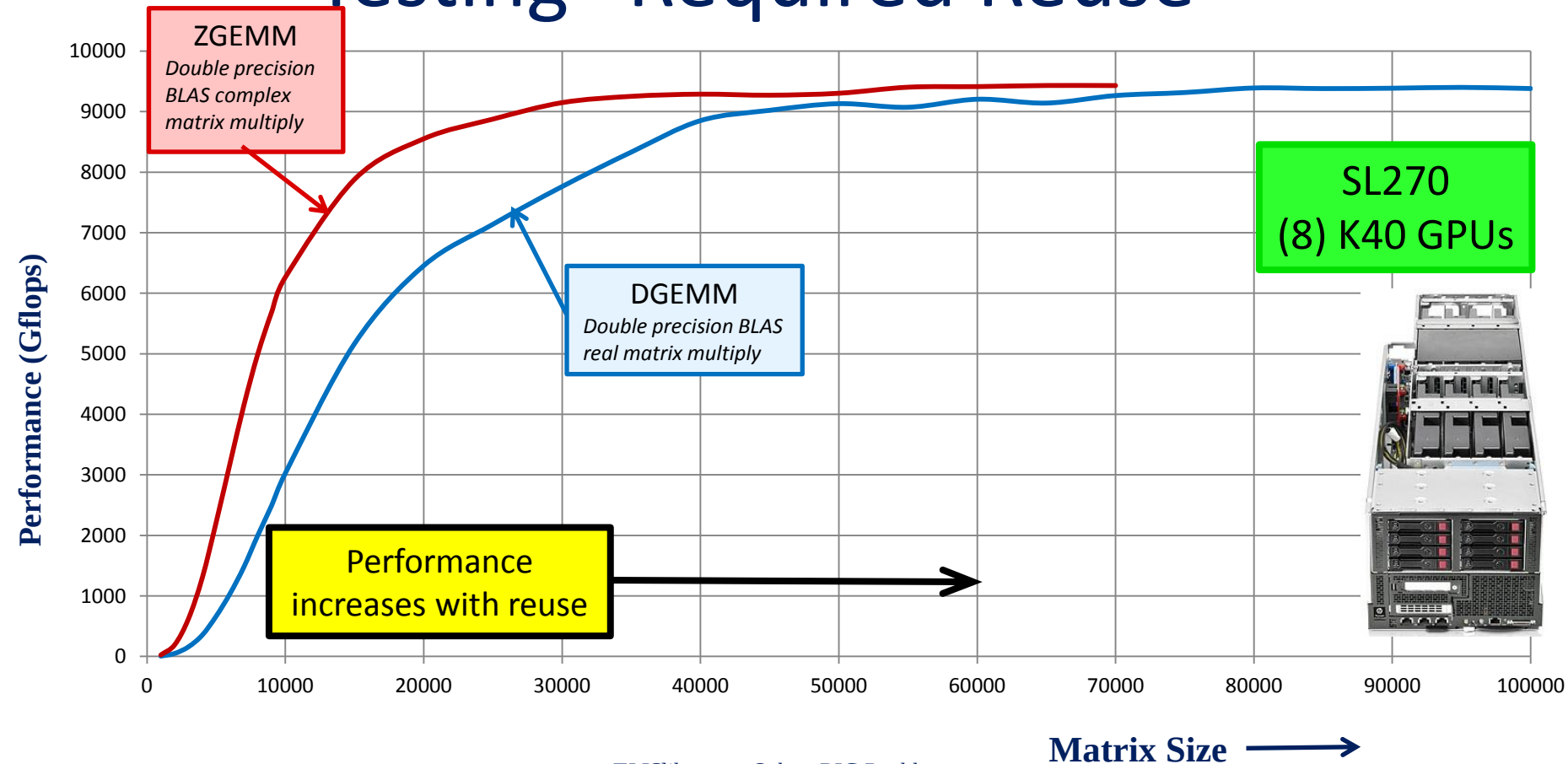
Testing “Required Reuse”



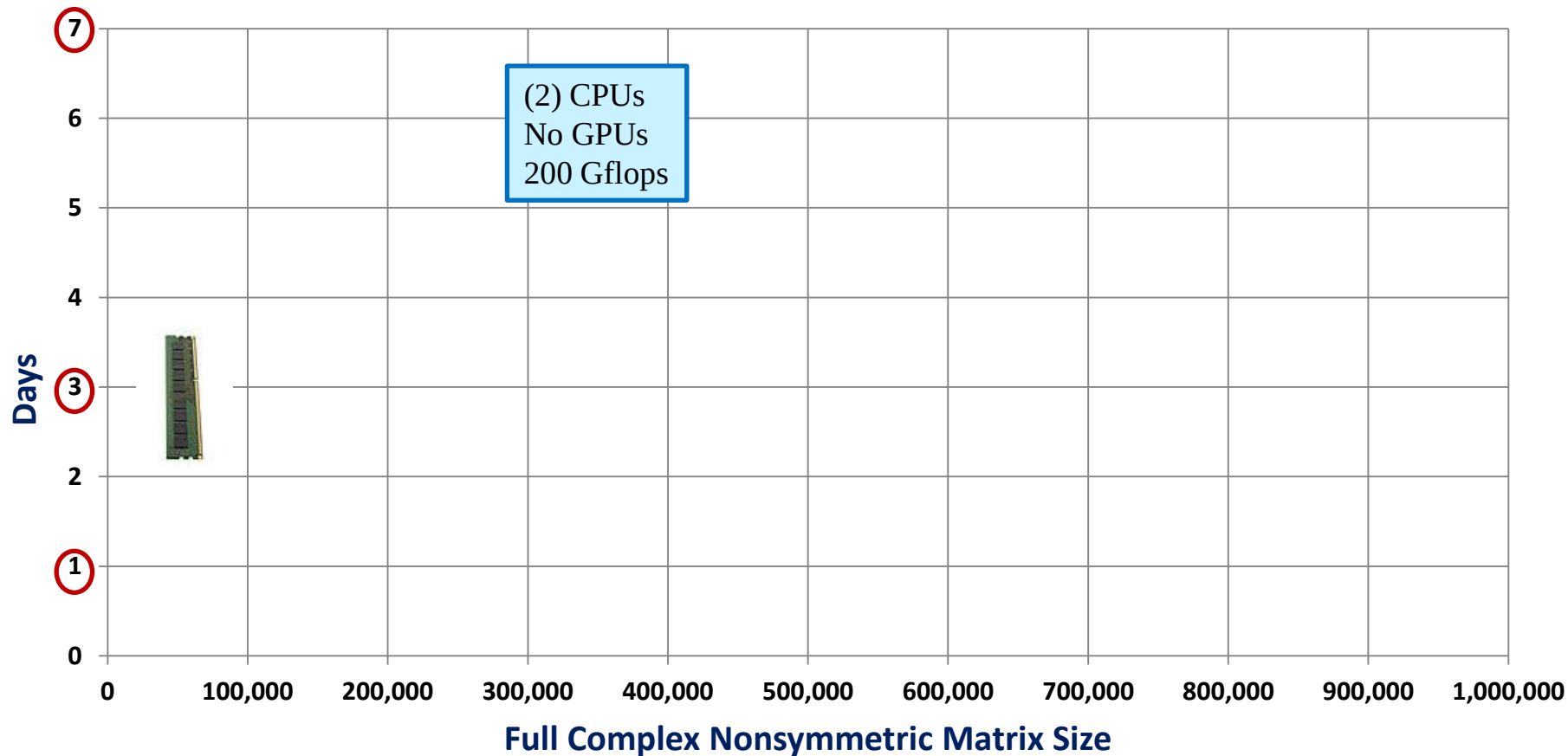
Testing “Required Reuse”



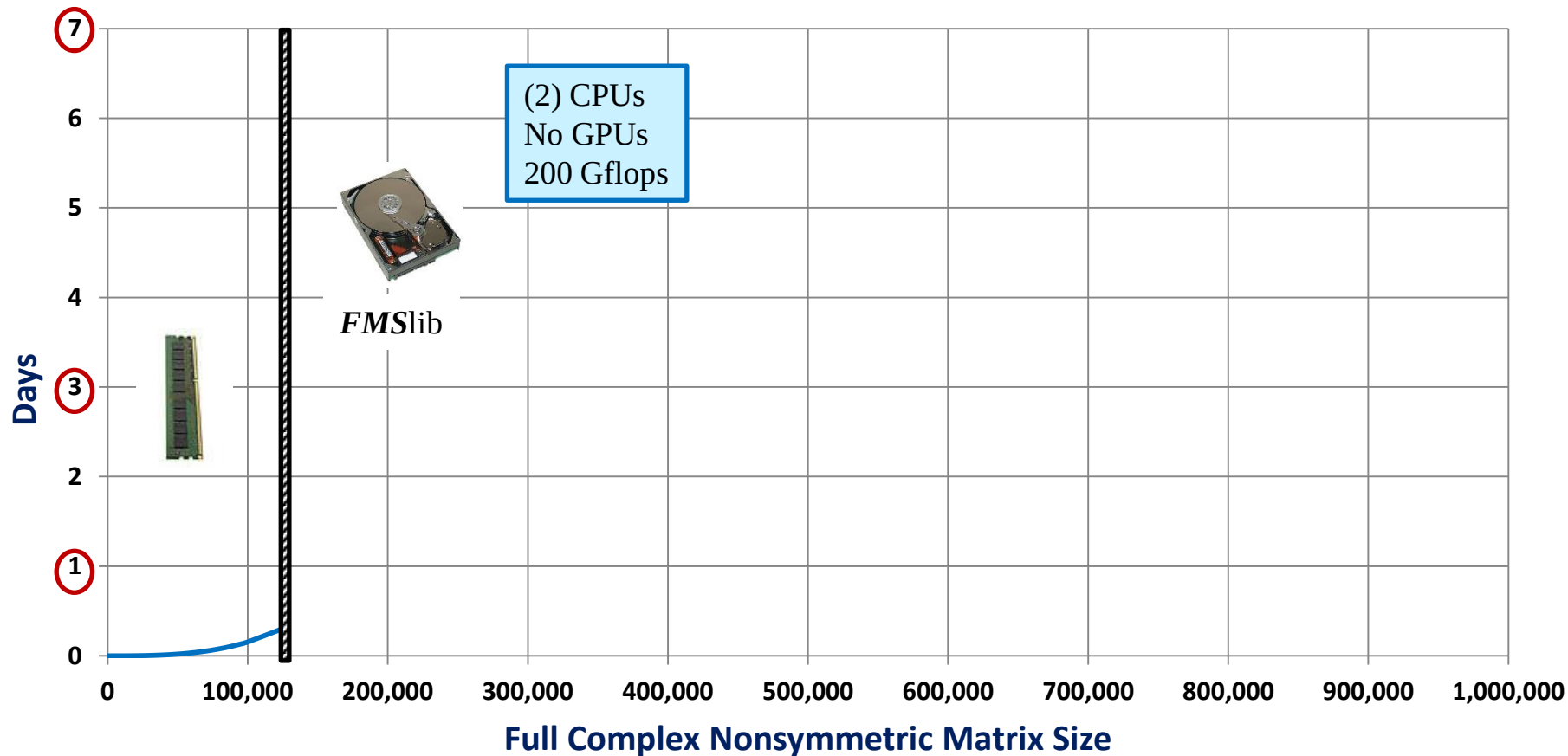
Testing “Required Reuse”



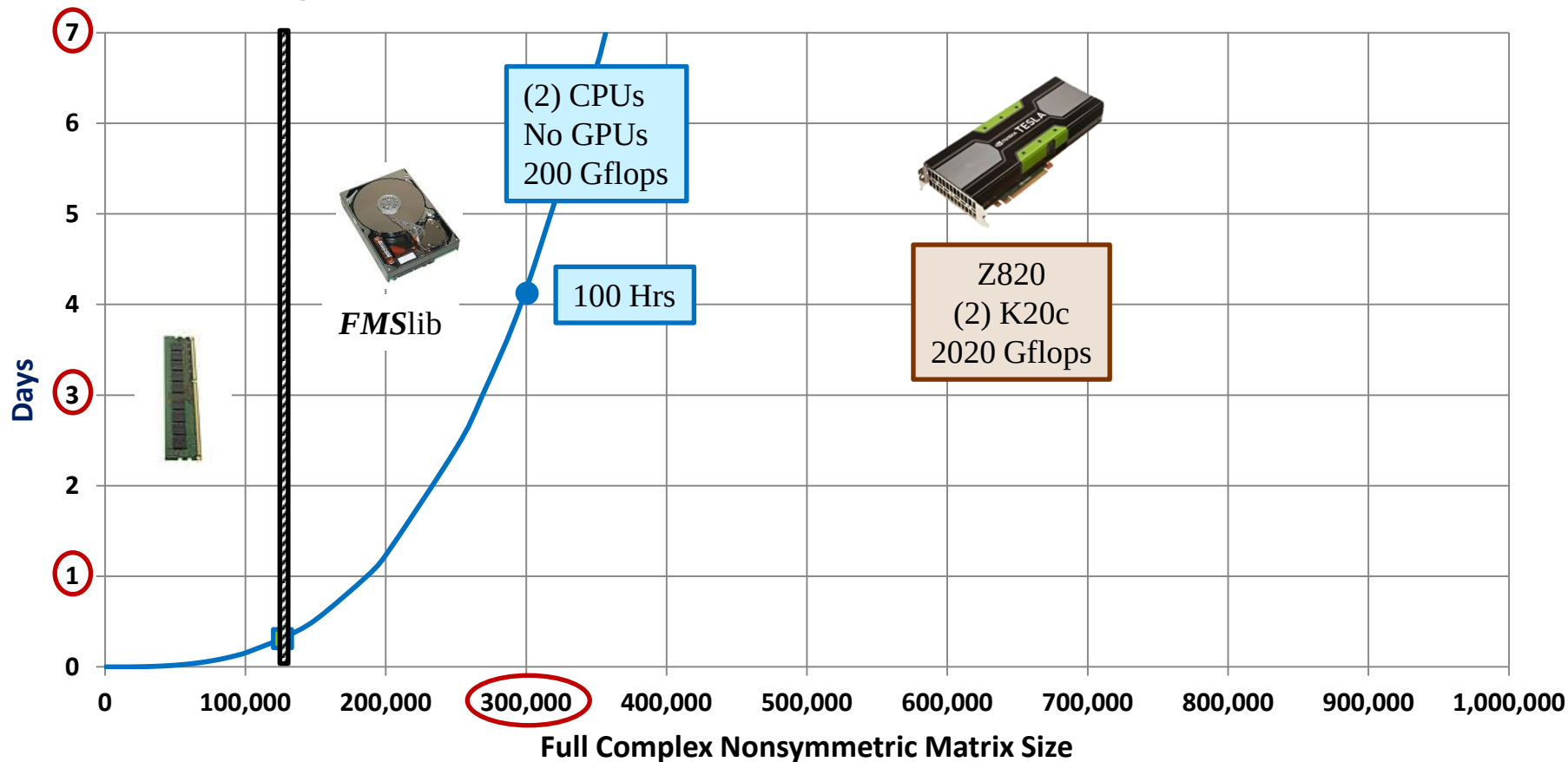
FMSlib Performance



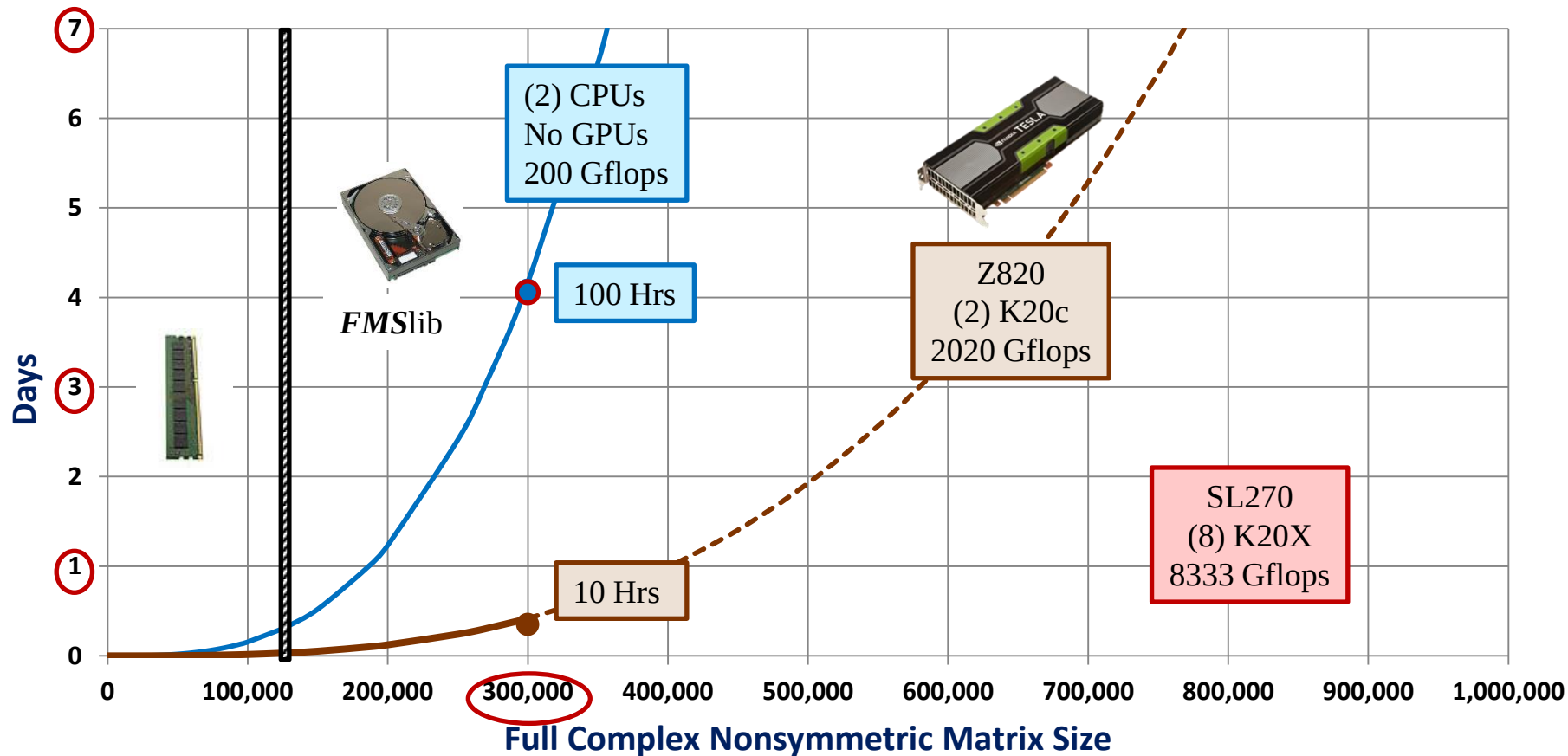
FMSlib Performance



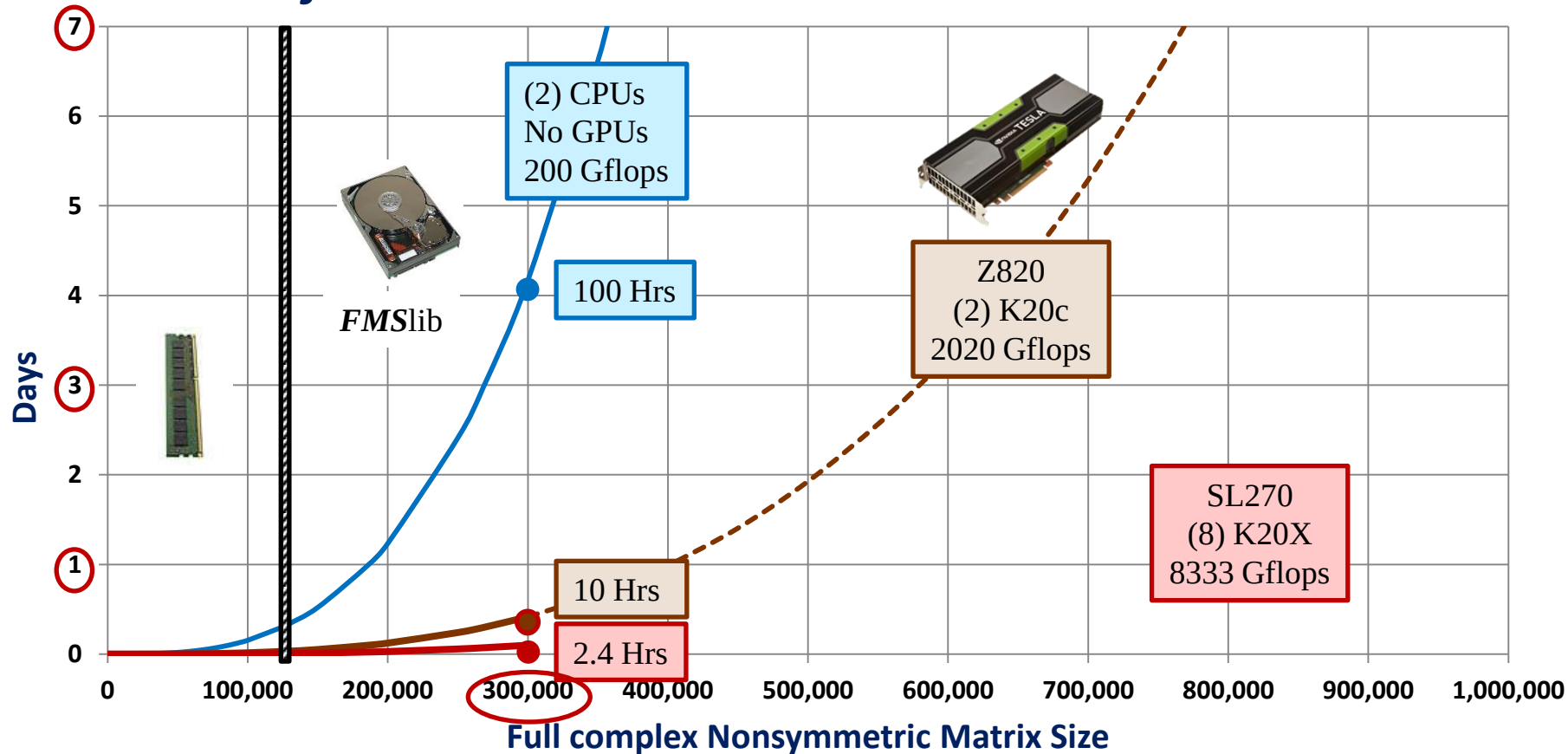
FMSlib Performance



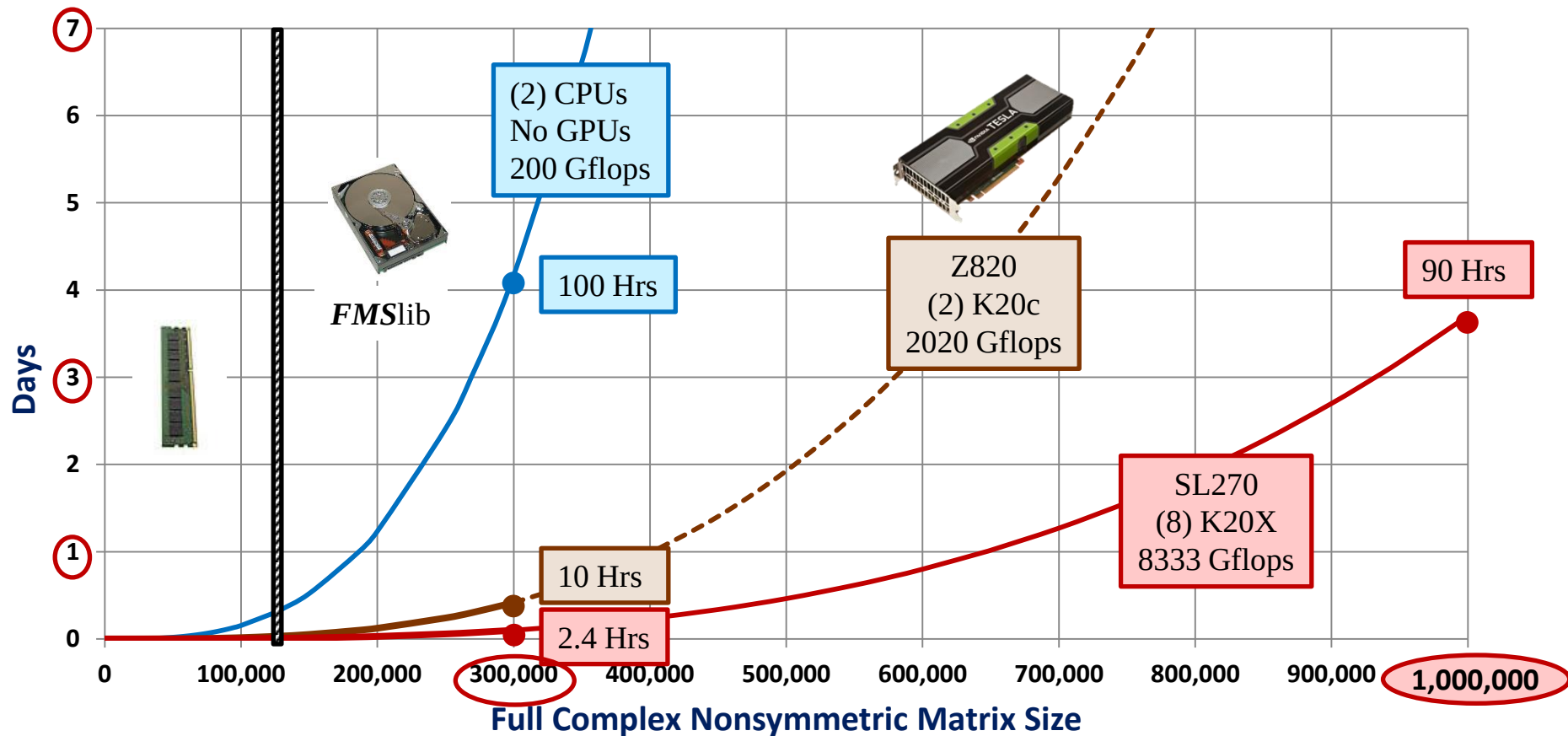
FMSlib Performance



FMSlib Performance



FMSlib Performance



Summary

- “REUSE” is required to achieve peak performance
- Implicit solutions provide high reuse
- Reuse can be implemented at a low level in hardware
- Floating point operations are inexpensive and getting cheaper.
- Data staging determines performance and cost

Next Steps

FREE

1. Download and run **[Matrix]Warrior**, an application which benchmarks and demonstrates FMSlib



- Form $[A]$, $\{B\}$ with test data
- Use FMSlib to solve $[A]\{X\}=\{B\}$
- Display reports as WEB pages

2. Install FMSlib in your application
3. Run FMSlib with data in memory
4. Test FMSlib with data on disc

When you are ready:

1. Purchase a system with GPUs (Z820, SL270)
2. License FMSlib

[Home](#) | [CPUs](#) | [GPU's](#) | [Memory](#) | [Disks](#) | [Files](#) | [Software](#) | [Cable's](#) | [Peripherals](#) | [Performance](#) | [Links](#) | [Help](#) | [Feedback](#)

[Matrix] Warrior Powered by FMSlib

Performance Analysis for Subroutine CNDF : Time used=24:57

COMPLETED				
17% IO	11% Other	44% MM	20% TR	9% DUA

Performance (Gflops)

Routine	All	CPU's	GPU 0	GPU 1	GPU 2	GPU 3	GPU 4	GPU 5	GPU 6	GPU 7
Matrix Multiply (CPU2N)	8707	291	1098	1104	1086	1108	1079	1086	1066	1091
Triangle Solve (CPU2N)	8315	235	1051	1054	1046	1065	1046	1054	1039	1047
Diagonal Factor	5796	7	774	795	765	756	726	713	687	677
Routine Overall	8331	270	1081	1086	1071	1091	1065	1072	1054	1073

Time (Sec.)

Routine	All	CPU's	GPU 1	GPU 2	GPU 3	GPU 4	GPU 5	GPU 6	GPU 7	GPU 8
Matrix Multiply:	671	600	645	642	652	640	657	653	665	650
Triangle Solve	312	262	301	300	302	297	302	300	304	302
Diagonal Factor	98	13	8	8	8	8	9	9	10	10
Total Count	1080	874	954	950	963	945	968	962	979	962
IO wait	267									
Other	150									
Overall	1497									

Times and Problem

Job Started	0	07:45:37 Sat Jan 07 2014	Equations	150000
Routine Started	131	07:47:49 Sat Jan 07 2014	Vectors	0
Current Time	1628	08:12:46 Sat Jan 07 2014	Block Size	38400 X 38400
Estimated Completion	1628	08:12:46 Sat Jan 07 2014	Data Type	Complex

IO Compute Ratio

Compute Rate (Gflops)	8311
Read Rate (MBytes/Sec)	809
Reuse	76800
IO Compute Ratio	0.91

Reuse (Disc->Memory) = 76800

Page updated 64 times. Automatically refreshed every 21 sec.: Last Updated 08:12:46 Sat Jan 07 2014

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[Matrix]Warrior

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Fast Matrix Solver



World's fastest solutions for

$$[A]\{X\} = \{B\}$$

Now Available Free

[Matrix] Warrior

MANUAL

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FMSLIB

Watch Presentations

NVIDIA GPU Technology Conference

Mar 21, 2013

Benchmark your GPUs with MatrixWarrior

May 17, 2012

Teraflop GPU Acceleration Of Large Matrix Algebra

DESCRIPTION

WHY FMS?

WHY GPUS?

WHY DISKS?

APPLICATIONS

BENEFITS

FEATURES

EASE OF USE

BENCHMARKS